Instituto de Física

Universidade Federal Fluminense

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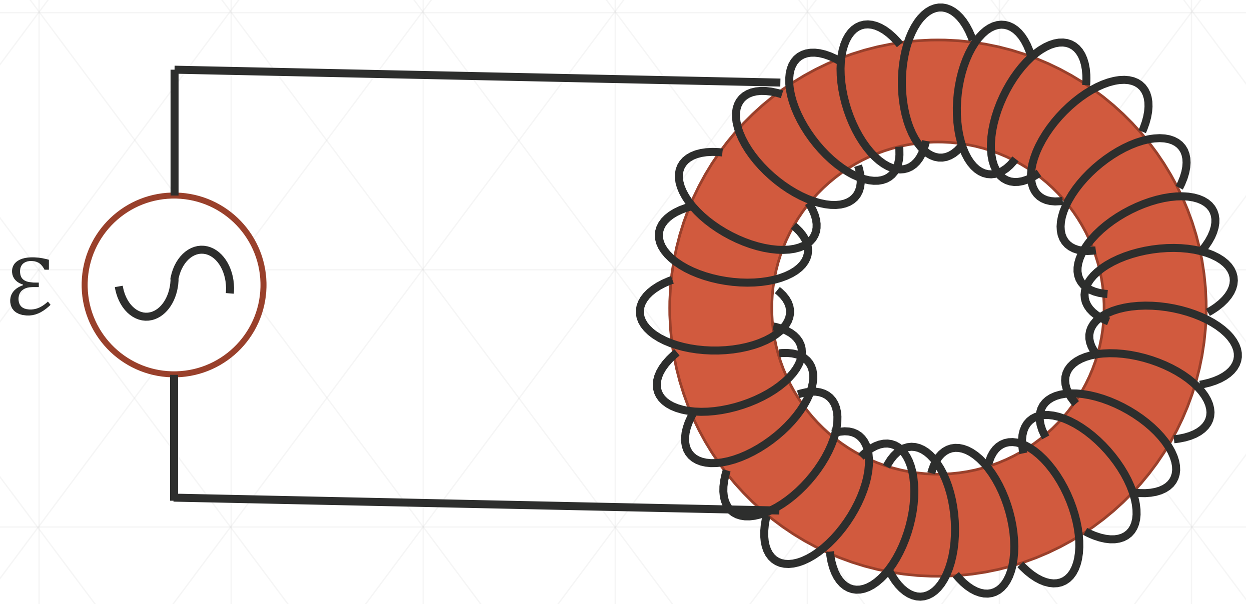
Eletromagnetismo

Newton Mansur



$$\mathcal{E} = Ri + \frac{d\Psi}{dt}$$

$$\mathcal{E}i = Ri^2 + i\frac{d\Psi}{dt}$$

$$\mathcal{E}idt = Ri^2dt + id\Psi$$

$$\Psi = N\phi = NBS$$

$$\mathcal{E}idt = Ri^2dt + iNSdB$$

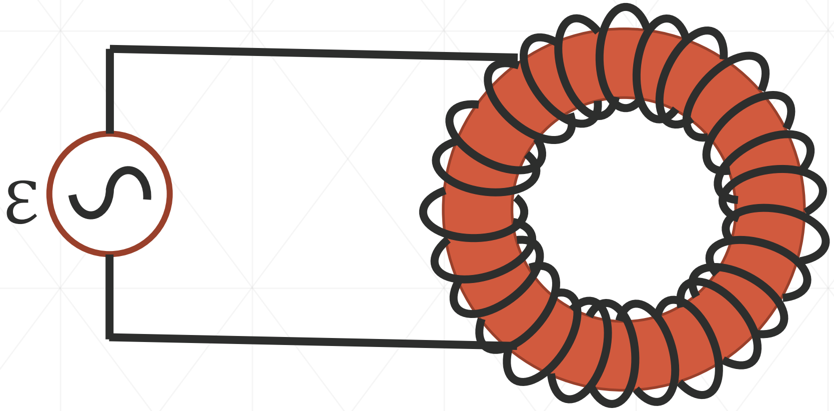
$$iNSdB \rightarrow d\left(\frac{1}{2}Li^2\right) = Lidi$$

$$H = ni = \frac{N}{l}i$$

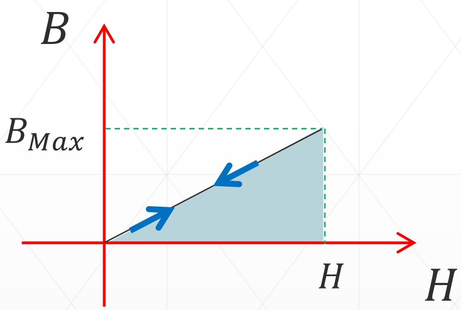
$$Ni = Hl$$

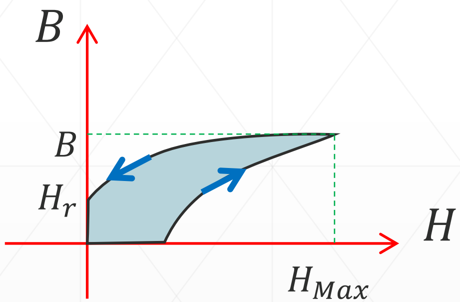
$$dW_m = HlSdB$$

$$W_m = V\int_0^{B_{Max}} HdB$$

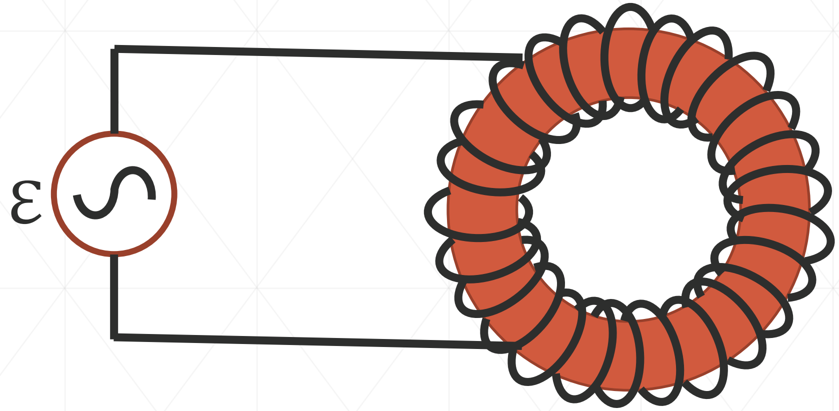


$$W_m = V\int_0^{B_{Max}} H\,dB \qquad H = \mu B \qquad W_m = \frac{1}{2}Li^2$$



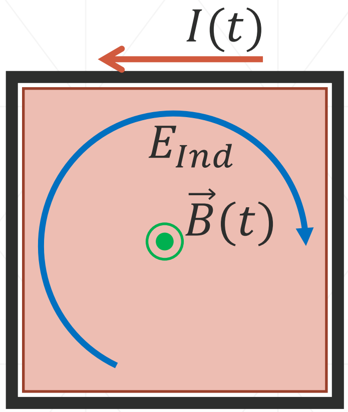


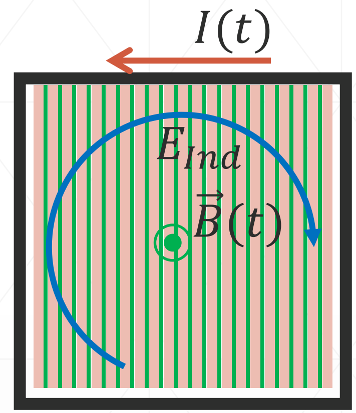
Os materiais ferromagnéticos *duros* têm ciclos de histerese com áreas grandes e dão origem a maiores perdas por histerese, não sendo adequados para a construção de máquinas eléctricas, em geral.



$$\Psi = N\phi = N\frac{NI}{R_m} = \frac{N^2}{R_m}I$$

$$L = \frac{N^2}{R_m}$$

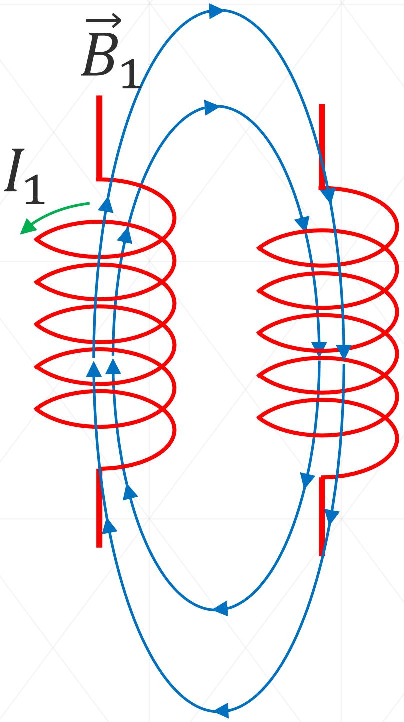




- Materiais não duros com baixa histerese e baixa relutância

- Construído em lâminas isolados com materiais isolantes evitando corrente de Foucault

Acoplamento magnético



$i_1 \rightarrow \phi_1 = \phi_{11} + \phi_{12}$

$i_2 \rightarrow \phi_2 = \phi_{22} + \phi_{21}$

$\Psi_{11} = N_1(\phi_{11} + \phi_{12}) = N_1\phi_1$

$\Psi_{22} = N_2(\phi_{22} + \phi_{21}) = N_2\phi_2$

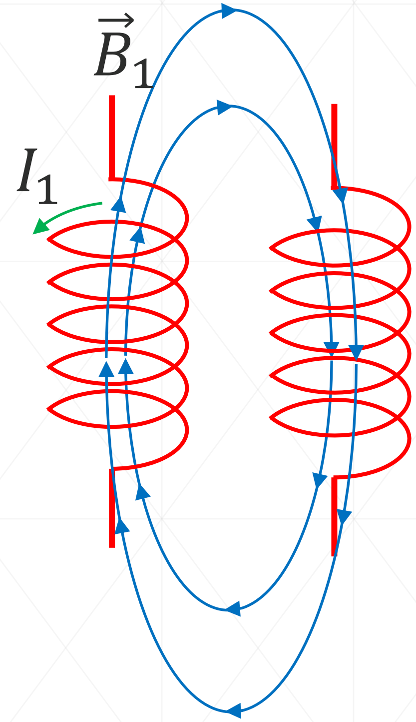
$\Psi_{1t} = N_1(\phi_1 \pm \phi_{21}) = \Psi_{11} \pm \Psi_{21}$

$\Psi_{2t} = N_2(\phi_2 \pm \phi_{12}) = \Psi_{22} \pm \Psi_{12}$

$\Psi_{1t} = L_{11}I_1 \pm M_{21}I_2$

$\Psi_{2t} = L_{22}I_2 \pm M_{12}I_1$

Acoplamento magnético



$$\Psi_{1t} = L_{11}I_1 \pm M_{21}I_2$$

$$\Psi_{2t} = L_{22}I_2 \pm M_{12}I_1$$

$$M_{12} = M_{21} = M$$

$$\mathcal{E}_{1t} = -\frac{d\Psi_{1t}}{dt} = -L_{11}\frac{dI_1}{dt} \mp M\frac{dI_2}{dt}$$

$$\mathcal{E}_{2t} = -\frac{d\Psi_{2t}}{dt} = -L_{22}\frac{dI_2}{dt} \mp M\frac{dI_1}{dt}$$

$$\mathcal{E}_{1t}I_1 = -\frac{d\Psi_{1t}}{dt} = -L_{11}I_1\frac{dI_1}{dt} \mp MI_1\frac{dI_2}{dt}$$

$$\mathcal{E}_{2t}I_2 = -\frac{d\Psi_{2t}}{dt} = -L_{22}I_2\frac{dI_2}{dt} \mp MI_2\frac{dI_1}{dt}$$

$$W_{mt} = \frac{1}{2}L_{11}{I_1}^2 + \frac{1}{2}L_{22}{I_2}^2 \pm M_{21}I_1I_2$$

Acoplamento magnético

$$\Psi_{1t} = L_1 I_1 + M I_2$$

$$\Psi_{2t} = L_2 I_2 + M I_1$$

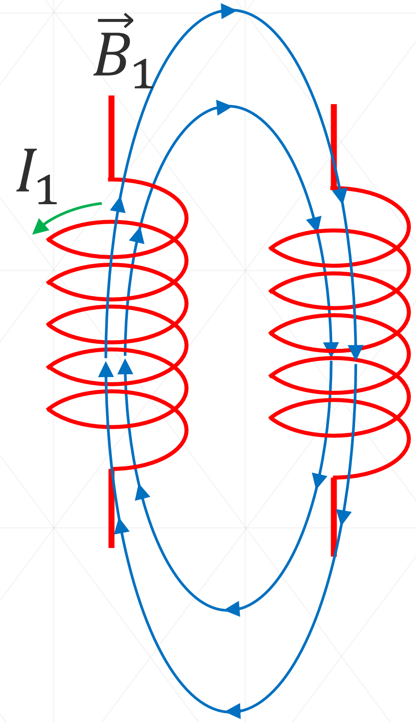
$$\begin{pmatrix} \Psi_{1t} \\ \Psi_{2t} \end{pmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{pmatrix} I_1 \\ I_1 \end{pmatrix}$$

$$\Delta = L_1 L_2 - M^2 = L_1 L_2 \left(1 - \frac{M^2}{L_1 L_2}\right)$$

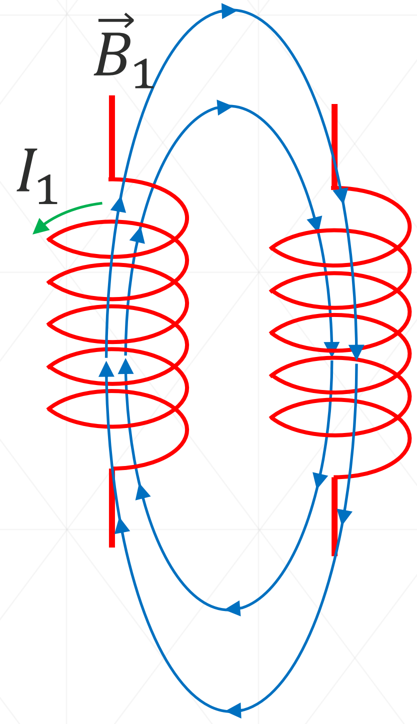
$$k = \sqrt{\frac{M^2}{L_1 L_2}} \quad \rightarrow \quad \textit{Coeficiente de acoplamento}$$

$$M = \sqrt{L_1 L_2} \quad \rightarrow \quad k = 1 \quad \textit{acoplamento ideal}$$

$$M = 0 \quad \rightarrow \quad k = 0 \quad \textit{sistema desacoplado}$$



Acoplamento magnético



$$\mathcal{E}_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$I_1 = i_1 e^{-j\omega t}$$

$$\mathcal{E}_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$I_2 = i_2 e^{-j\omega t}$$

$$\mathcal{E}_1 = j\omega(L_1 I_1 + M I_2)$$

$$\mathcal{E}_2 = j\omega(L_2 I_2 + M I_1)$$

$$L_1 \propto {N_1}^2$$

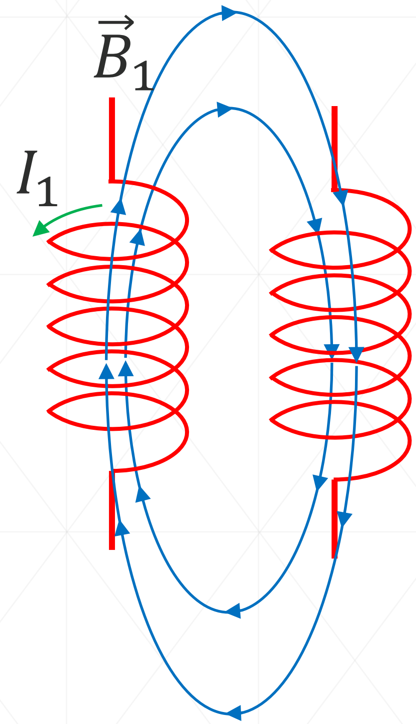
$$L_2 \propto {N_2}^2$$

$$\frac{L_1}{L_2} = \frac{{N_1}^2}{{N_2}^2}$$

$$k^2 = \frac{M^2}{L_1 L_2} \qquad L_1 L_2 = \frac{M^2}{k^2} \qquad \frac{L_1}{M} = \frac{N_1}{N_2 k} \qquad \frac{L_2}{M} = \frac{N_2}{N_1 k}$$

$$\frac{L_1}{M}\mathcal{E}_2 = j\omega M\left(\frac{L_1 L_2}{M^2} I_2 + \frac{L_1}{M} I_1\right) \qquad \mathcal{E}_1 = j\omega M\left(\frac{L_1}{M} I_1 + I_2\right)$$

Acoplamento magnético



$$\mathcal{E}_1 = \frac{L_1}{M}\mathcal{E}_2 + j\omega M\left(1 - \frac{L_1 L_2}{M^2}\right) I_2$$

$$\mathcal{E}_1 = \frac{N_1}{N_2 k}\mathcal{E}_2 + j\omega M\left(1 - \frac{1}{k^2}\right) I_2$$

$$I_1 = \frac{1}{j\omega M}\mathcal{E}_2 - \frac{N_2}{N_1 k} I_2$$

Acoplamento ideal $\rightarrow$ $k = 1$

$$\mathcal{E}_1 = \frac{N_1}{N_2}\mathcal{E}_2$$

$$I_1 = \frac{1}{j\omega M}\mathcal{E}_2 - \frac{N_2}{N_1} I_2$$

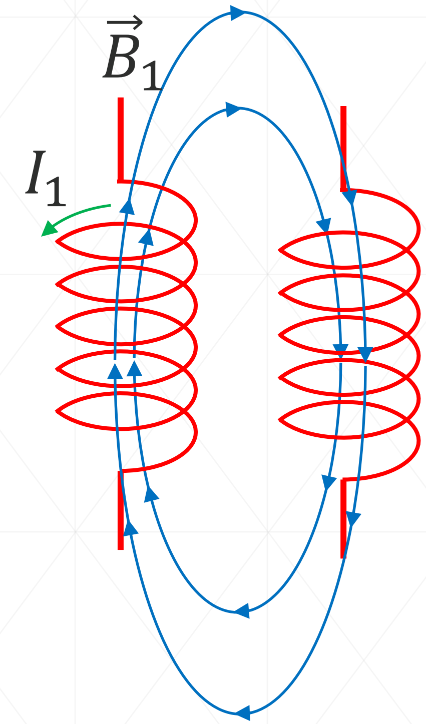
$\mathcal{E}_2 = 0$ $\rightarrow$ *Curto no secundário* $\quad I_1 = -\frac{N_2}{N_1} I_2$

$\frac{1}{j\omega M}\mathcal{E}_2$ $\rightarrow$ *Corrente de magnetização*

Acoplamento magnético

$$\mathcal{E}_1 = j\omega(L_1 I_1 + M I_2)$$

$$\mathcal{E}_2 = j\omega(L_2 I_2 + M I_1)$$



$$\mathcal{E}_1 = R_1 I_1 + j\omega(L_1 I_1 + M I_2)$$

$$\mathcal{E}_2 = R_2 I_2 + j\omega(L_2 I_2 + M I_1)$$

$$\begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{pmatrix} I_1 \\ I_1 \end{pmatrix} + j\omega \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix} = \begin{bmatrix} R_1 + j\omega L_1 & j\omega M \\ j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} R_1 + j\omega L_1 & j\omega M \\ j\omega M & R_2 + j\omega L_2 \end{bmatrix}^{-1} \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}$$

Equações de Maxwell

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Em 1 dimensão

$$\frac{\partial^2 E_x(x,t)}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x(x,t)}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 E_x(x,t)}{\partial t^2}$$

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0}} = 2{,}998x10^8 \frac{m}{s} = c$$

$$\frac{\partial^2 E_x(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 E_x(x,t)}{\partial t^2}$$

$$E_x(x,t) = X(x)T(t)$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} = \frac{1}{Tv^2}\frac{d^2 T}{dt^2} = -\beta^2$$

$$T(t) = T_0 e^{-j\beta v t}$$

$$X(x) = X_0 e^{\pm j\beta x}$$

$$E_x(x,t) = E_0 e^{-j\beta(\pm x + vt)}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta v = \omega = \frac{2\pi}{T}$$

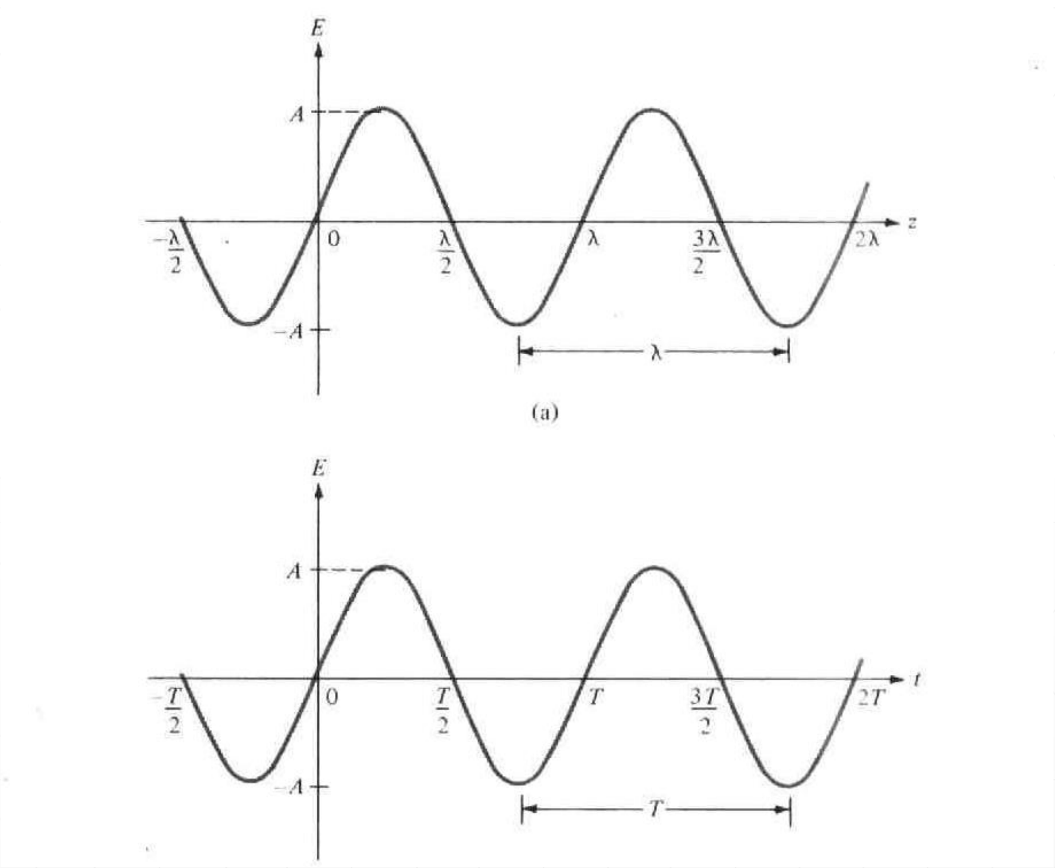
$$\vec{E}(\vec{r},t) = E_0 e^{-j\,(\pm\vec{\beta}.\vec{r} + \omega t)}$$

$$\vec{E}(\vec{r},t) = E_0 e^{-j\,(\vec{k}.\vec{r} \pm \omega t)}$$

$$E_x(x,t) = E_0 e^{-j\,(\pm\beta x+\omega t)}$$

$$\beta = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

$$v = \frac{\omega}{\beta}$$



Condutor com condutividade baixa (dielétrico com perda)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \vec{J} = \sigma \vec{E}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Sem carga livre e $\vec{E}\,(\vec{r}, t) = \vec{E}_0\,(\vec{r}) e^{j\omega t}$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$$

$$\vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H}$$

Condutor com condutividade baixa (dielétrico com perda)

$$\nabla^2 \vec{E} = j\omega\mu(\sigma + j\omega\epsilon)\vec{E}$$

$$\frac{\partial^2 E_x(x,t)}{\partial x^2} = j\omega\mu(\sigma + j\omega\epsilon)E_x(x,t)$$

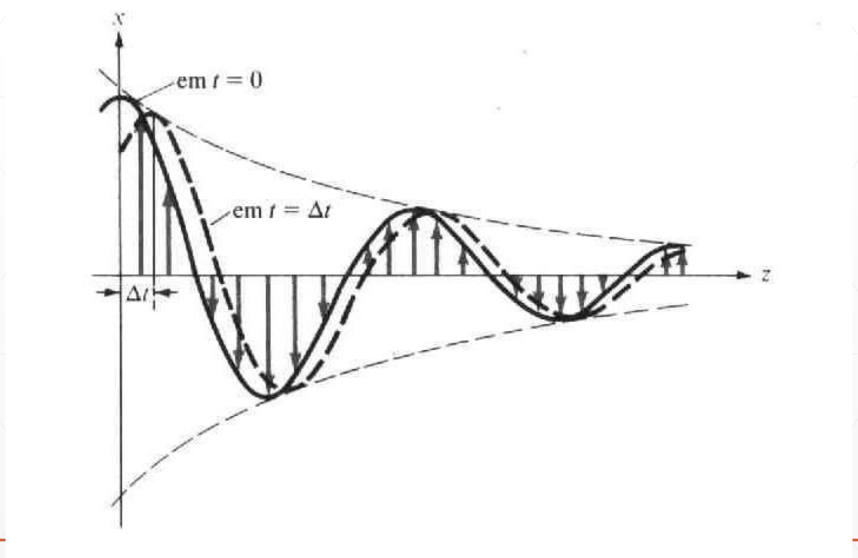
$$E_x(x,t) = E_0 e^{-j\,(\pm\gamma x + \omega t)}$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$E_x(x,t) = E_0 e^{-\alpha x}\cos(\omega t \pm \beta x)$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}$$

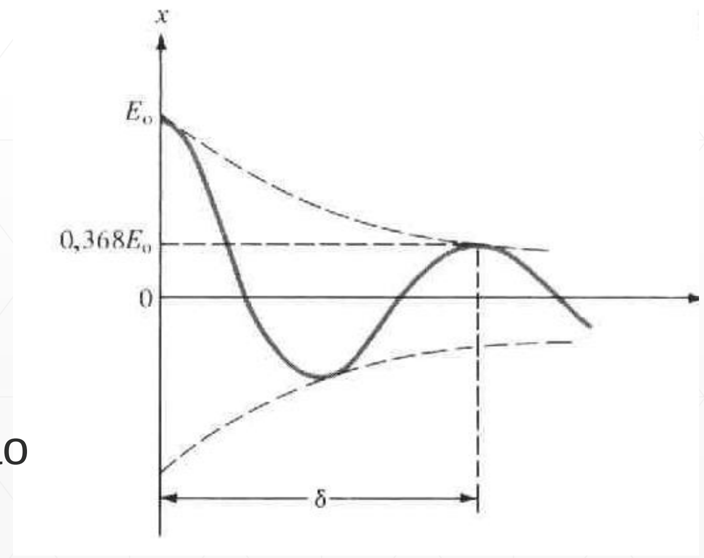


Condutor com condutividade alta (condutor quase ideal)

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1} = \sqrt{\frac{\mu\omega\sigma}{2}} \qquad \beta = \omega \sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1} = \sqrt{\frac{\mu\omega\sigma}{2}}$$

$$E_x(x,t) = E_0 e^{-\alpha x}\cos(\omega t \pm \beta x) = E_0 e^{-x/\delta}\cos(\omega t \pm \beta x)$$

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}}$$



skin depth

Profundidade de penetração

$Dielétrico\ sem\ perda \qquad \sigma = 0$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1} = 0$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1} = \omega\sqrt{\mu\epsilon}$$

Meio dispersivo

$$\beta = \omega\sqrt{\mu_0\epsilon(\omega)}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\epsilon(\omega)}}$$